Time and Frequency Basics :

oscillators and clocks

A sinusoidal signal (or any periodic signal as we know that a periodic signal might be described by a truncated sum of sinusoidal signals), is describe in terms of amplitude, frequency and phase. In time and frequency domain we often use :

$$V(t) = A(t).\cos(2\pi\vartheta(t)t) = A(t)\cos(\varphi(t)) = A(t)\cos(2\pi\vartheta_0 T(t))$$

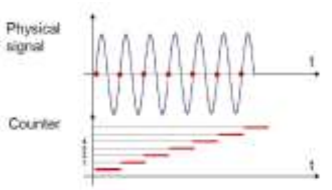
which allows to descried time variation of the signal in terms of frequency variation, phase variation or elapsed time variation. We then introduce the definition of the instantaneous frequency $\vartheta(t) = \frac{1}{2\pi}\frac{\delta\varphi(t)}{\delta t}$, the instantaneous phase : $\varphi(t) = 2\pi\int_o^t \vartheta(t')dt'$, and the instantaneous time $T(t) = \frac{\varphi(t)}{2\pi\vartheta_0} = \frac{\varphi(t)}{2\pi}.\frac{1}{\vartheta_0}$, where $\frac{\varphi(t)}{2\pi}$ is the "number of cycles" observed since t_0, when counting began, and $\frac{1}{\vartheta_0}$ is the period (time duration of one cycle) of the periodic signal.

One can also express, mainly when dealing with high stability sources where we will be more interested by small variation (noise) of frequency, the instantaneous frequency by $\vartheta(t) = \vartheta_0 .(1 + \epsilon + y(t))$, where ϑ_0 is the ideal frequency, ϵ is the systematic (deterministic) drift affecting the frequency and $y(t) = \frac{\vartheta(t)-\vartheta_0}{\vartheta_0} = \frac{\delta\vartheta(t)}{\vartheta_0}$ is the relative frequency variation, which allow to define instantaneous phase as $\varphi(t) = 2\pi\int_0^t \vartheta(t')dt' = 2\pi\vartheta_0[(1+\epsilon).t + \int_0^t y(t')dt']$ and $T(t) = \frac{\varphi(t)}{2\pi\vartheta_0} = (1+\epsilon) + x(t)$, with $x(t) = \int_0^t y(t')dt' <=> y(t) = \frac{dx(t)}{dt}$.

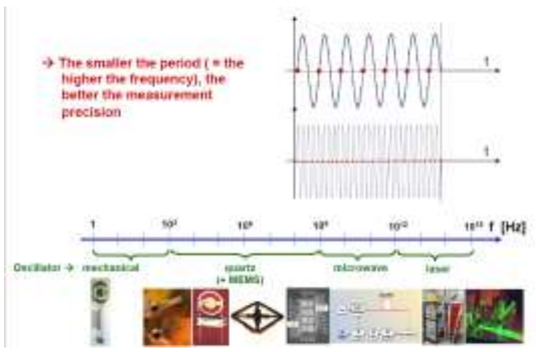
The characterization of oscillators and clocks will be then to identify the frequency variation (f-Fourier spectrum) of y(t), providing $S_y(f)$ (in Hz^{-1}), of $\nu(t)$ providing $S_\nu(f)$ (in $Hz^2.Hz^{-1}$] and $\sigma_y(t)$ in frequency domain, or $\phi(t)$ providing $S_\phi(f)$ (in $rad^2.Hz^{-1}$) in phase domain or $\sigma_x(t)$ (in s) in time domain.

Oscillators are devices providing periodic event, movement or energy state, allowing a signal to reproduce itself every period or to oscillate between two equilibrium states.

Then a clock, as depicted below is nothing else than a count of period since the origin…



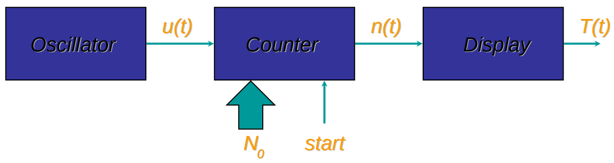
High frequency, smaller period, greater number of periods to count the same event, better accuracy. Resonators physic might be mechanical (pendulum) acoustic wave (piezo, MEMS,) RF wave (atomic transition, RF wave propagation) , optical (laser, electronic transition, optical wave delay : fiber , WGM,…)



Noises and deterministic drift affecting oscillators and clocks, will need characterization in each of the user domain, frequency, phase or time.

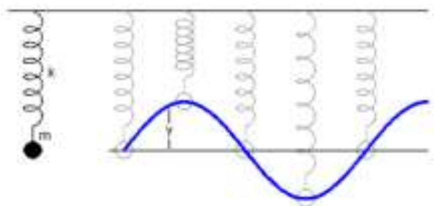
A **clock** is then made of :

- A periodic **oscillator**, providing known cycle duration, t_u
- A mean to **count** the number of periods, N(t)
- A mean to **re initiate** the counting
- A definition of **time origin**, t_o



Resonators might be based on mechanical , acoustic wave propagation and resonant condition, on quantum state population/depopulation, on optical resonances ,…

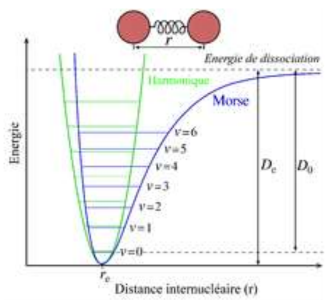
Mechanical harmonic Oscillator



- $F = -kx$
- $F = m.\frac{d^2x}{dt^2}$
- $x(t) = x_0.\cos(\omega t + \varphi)$
- $k.x(t) = m.x_0.\omega^2.\cos(\omega t + \varphi)$
- $k.x(t) = m.\omega^2.x(t)$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Quantum harmonic oscillator

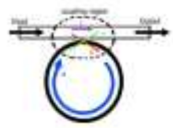


$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

$$\hat{H}.|\varphi> = E.|\varphi>$$

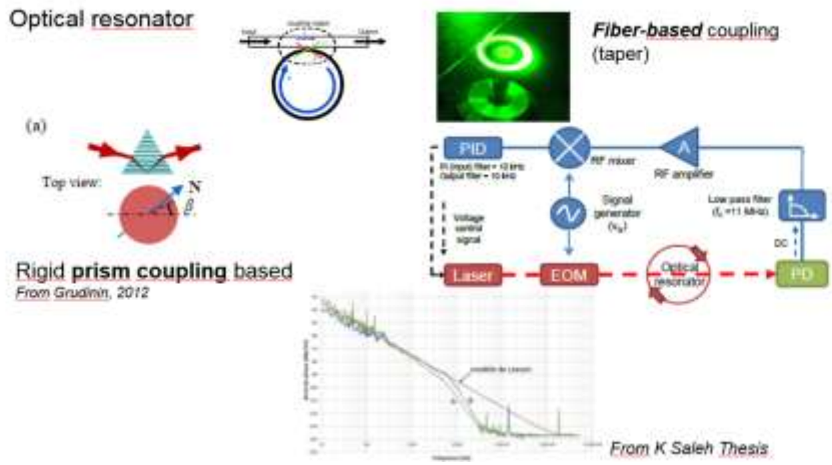
$$E_v = \hbar\omega\left(v + \frac{1}{2}\right)$$

Resonators are based on mechanical vibrations (acoustic waves…), sustained resonances between states, RF or optical waves constructive interferences (resonances).

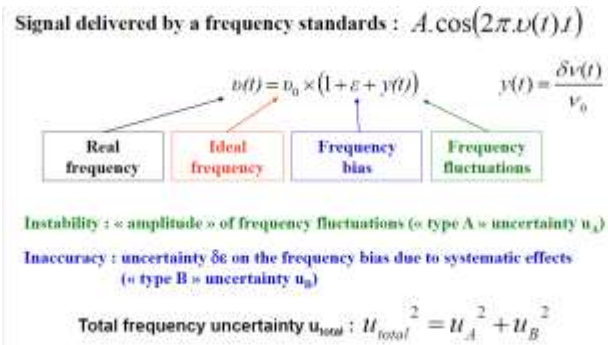
Acoustic	Quantum based	Optical
		
$Q = \vartheta/\delta\vartheta$	$Q = \vartheta/\delta\vartheta$	$Q = \vartheta/\delta\vartheta$
ν: kHz to GHz	ν: GHz to 10's GHz	ν: # 100's THz
δν: # Hz- kHz	δν: # 10's-100'sHz	δν: # 10's 100's kHz
Q: 10^4 -10^6	Q: 10^7 -10^9	Q: 10^8-10^{10}

More precisely, resonators can be of resonant type or delay line, such as in SAW devices or fiber based resonators, where the time required to propagate the signal along a distance X between in and out ix given by X = V*t, where V might be an surface acoustic wave propagation velocity (typ. Some 1’000 m/s) , over a mm scale device, or light speed over fiber (km length range).

Optical resonators are gaining must interest since a decade, because they might be fiber-based (providing to day the best noise floor) or WGM-resonators based (Whispering Galery mode resonators) on crystalline or amorphous discs, from mm down to 10's of microns diameter size, using fiber coupling (taper) or prism coupling providing miniaturization and wafer scale integration capabilities.



As described above, the characterization of clock performances, either from a mm scale or from 100's liter size, need to quantify the fluctuation behavior in frequency, phase or time domain:

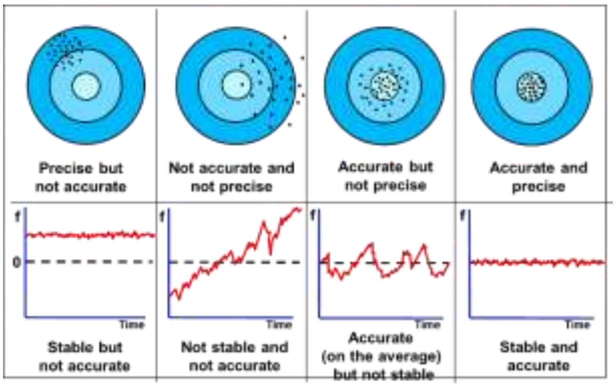


Deterministic drift (ie drift one can quantify and relay to an external variable , such ageing, sensitivity to environment (magnetic field, temperature, acceleration, …) and phase fluctuation impact gives:

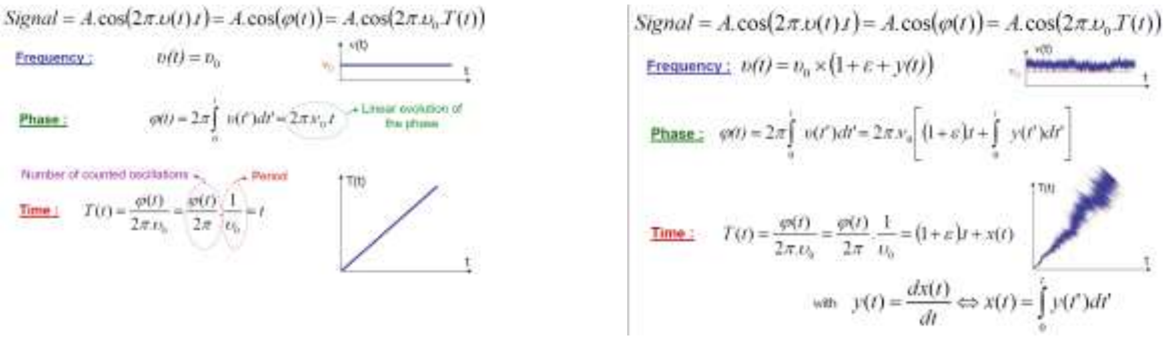
$$x(t) = x_0 + y_0\, t + \frac{D}{2}\, t^2 + \frac{\phi\,(t)}{2\,\pi\,\nu_0} + \int S_p(t)\, P(t)\, dt$$

Where x_0 is the initial phase-time offset, y_0 the initial relative frequency offset, D is the frequency drift with time (in 10^{-n} per unit of time), $S_p(t)$ is the sensitivity of frequency versus perturbation P(t), and P(t) is the time spectrum of application of the perturbation P. First part of equation is initial time and frequency offset and frequency drift, 2nd part in noise contribution, 3rd part is environmental.

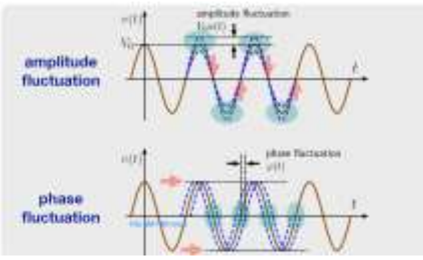
Oscillators and clocks are defined by their instabilities and noise, in frequency, phase and time domain. On can also describe a clock behavior in terms of accuracy and stability, by the following well known picture, due to J Vig:

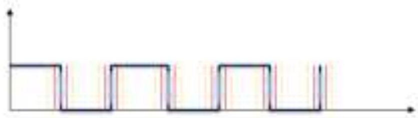


The behavior of an ideal clock and of a real one are given by this graph :



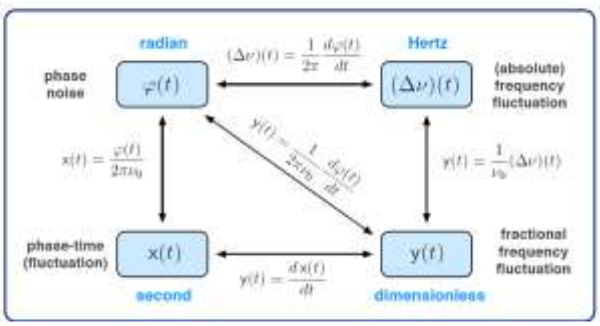
The oscillating signal might be affected by amplitude noise or phase noise,



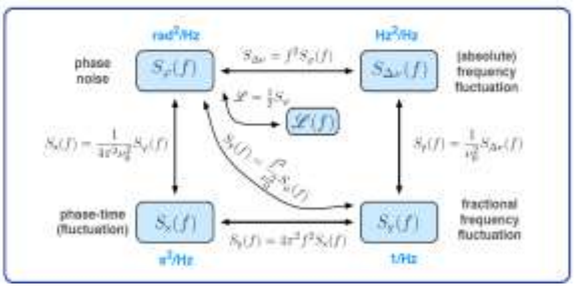


Time jitter

and all these characterizations are related to each other, and can be expressed one from each other. Choosing to perform instabilities characterization in frequency, phase or time domain, is only a matter of convenience, from experimental side, or from system area.



The physical relation between frequency, phase and time, and their instabilities characterization.



Phase noise ($S_\phi(f)$ and time variation ($\sigma_y(\tau)$) are probably the most commonly used characterization tools. The phase noise of oscillators is usually described in terms of Leeson model, giving the Fourier spectrum of variation under : $S_y(f) = \sum_i \frac{h_i}{f^i}$, or in $S_\phi(f)$ through $S_\phi(f) = \left(\frac{\vartheta_0}{f}\right)^2 S_y(f)$. And filtering noise impact by the loaded Q of the resonator:

$$\nu(t) = \nu_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = \text{"instantaneous" frequency;} \quad \phi(t) = \phi_0 + \int_0^t 2\pi[\nu(t') - \nu_0]dt'$$

$$y(t) = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0} = \text{normalized frequency;} \quad \phi^2_{RMS} = \int S_\phi(f)df$$

$$S_\phi(f) = \frac{\phi^2_{RMS}}{BW} = \left(\frac{\nu_0}{f}\right)^2 S_y(f); \qquad \mathcal{L}(f) = 1/2\, S_\phi(f), \text{ per IEEE Standard 1139-1988}$$

$$\sigma_y^2(\tau) = 1/2 < (\bar{y}_{k+1} - \bar{y}_k)^2 > = \frac{2}{(\pi\nu_0\tau)^2}\int_0^\infty S_\phi(f)\sin^4(\pi f\tau)df$$

The five common power-law noise processes in precision oscillators are:

$$S_y(f) = \underset{\text{(White PM)}}{h_2 f^2} + \underset{\text{(Flicker PM)}}{h_1 f} + \underset{\text{(White FM)}}{h_0} + \underset{\text{(Flicker FM)}}{h_{-1} f^{-1}} + \underset{\text{(Random-walk FM)}}{h_{-2} f^{-2}}$$

$$\text{Time deviation} = x(t) = \int_0^t y(t')dt' = \frac{\phi(t)}{2\pi\nu}$$

Measure	Symbol
Two-sample deviation, also called "Allan deviation"	$\sigma_y(\tau)$*
Spectral density of phase deviations	$S_\phi(f)$
Spectral density of fractional frequency deviations	$S_y(f)$
Phase noise	$\mathcal{L}(f)$*
* Most frequently found on oscillator specification sheets	

$$f^2 S_\phi(f) = \nu^2 S_y(f); \quad \mathcal{L}(f) = ½\,[S_\phi(f)] \qquad \text{(per IEEE Std. 1139),}$$

and

$$\sigma_y^2(\tau) = \frac{2}{(\pi\nu\tau)^2}\int_0^\infty S_\phi(f)\sin^4(\pi f\tau)df$$

Where τ = averaging time, ν = carrier frequency, and f = offset or Fourier frequency, or "frequency from the carrier".

The so-called Leeson formula, S_ϕ and S_φ are resonator and oscillator phase noise spectra, is then:

$$S_\varphi(f) = \left[1 + \frac{1}{f^2}\left(\frac{\nu_0}{2Q}\right)^2\right] S_\phi(f) \qquad \text{Leeson formula}$$

and, depending of the relative position of the cut-off frequency of loaded Q device and the flicker noise corner of the sustaining electronics, the Leeson noise spectrum comes out as:

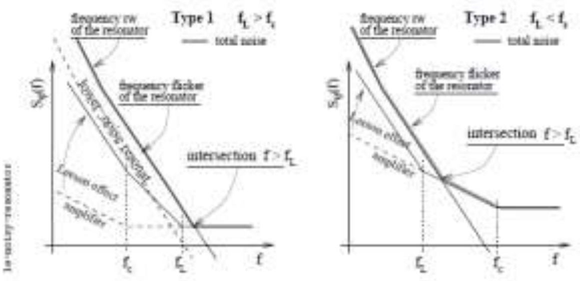
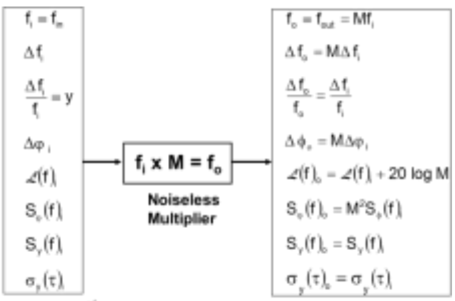
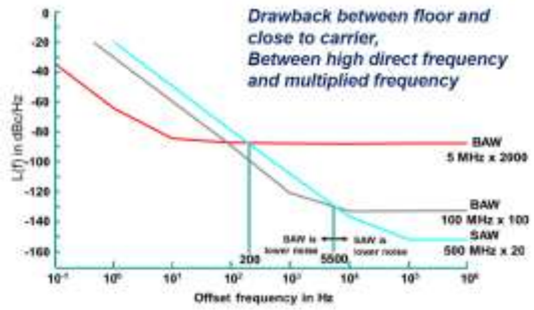


Figure 3.14: Effect of the resonator frequency fluctuations on the oscillator noise.

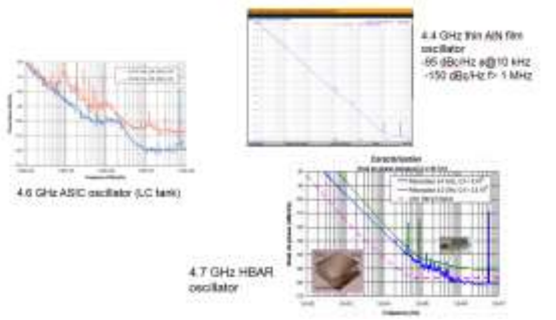
Application are going more and more towards RF frequencies, and the target is actually systems operating in the 30-90 GHz range. Options are then direct frequency sources (more optical based) or frequency multiplication. Drawback between direct or synthesized sources are estimated form noise spectrum contribution. Direct high frequency generally exhibit lower noise floor and wider close to carrier noise, while multiplied sources may exhibit better (narrower) close to carrier node and worst noise floor:

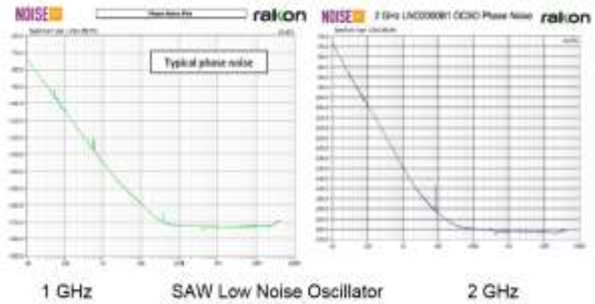


Note that $y = \frac{\Delta f}{f}$, $S_y(f)$, and $\sigma_y(\tau)$ are unaffected by frequency multiplication.



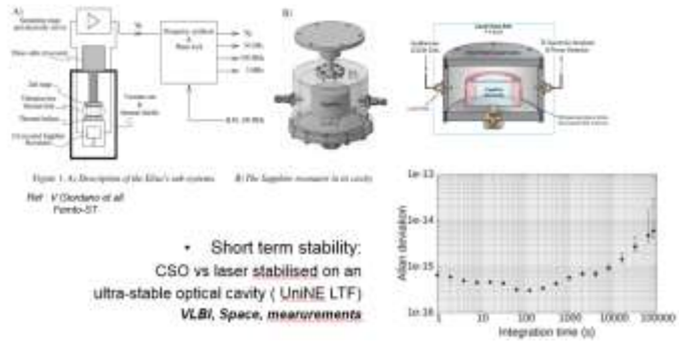
Noise of HF sources : phase noise spectra of high end MEMS devices (AlN, HBAR), ASIC in the 4-5 GHz range and 1-2 GHz direct frequency SAW Oscillators



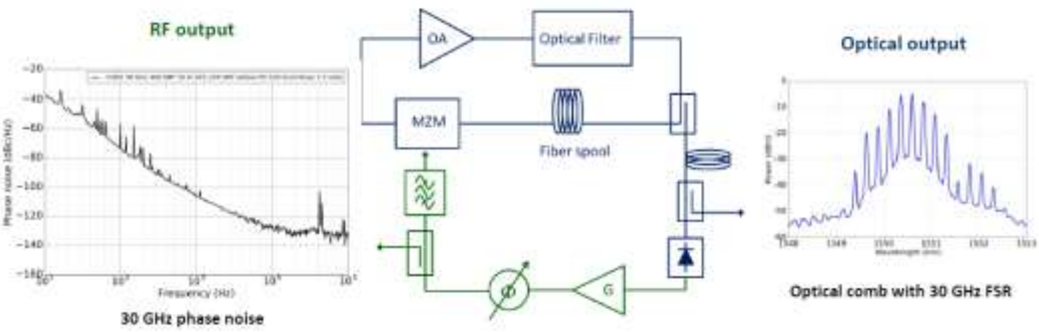


Such SAW device performances are actually the best compromise frequency / close to carrier noise, and they can be used in high rank multiplication to address 10-20 GHz range.

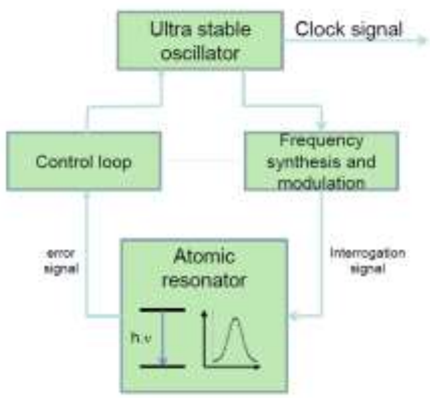
Around 10 GHz, the best short term stability is actually provided by Cryogenic sapphire



In the frequency range 10 GHz – 100 GHz, the actual solutions are mainly DROs, YIG oscillators, and optical oscillators, either WGM (Whispering Gallery mode) of fiber based resonators. An interesting fiber based topology, described by LAAS, is the COEO, coupled Optoelectronic Oscillator, and the start of the art performance is given in the next curve, showing the comb present at the optical output, and the low noise signal on the RF output:



In atomic clocks, such as H Maser, Cesium beam (either magnetic selection or optically pumping) a local oscillator is generally used to provide, through modulated HF synthesis, the RF interrogation of the physical device and the output of the atomic clock.

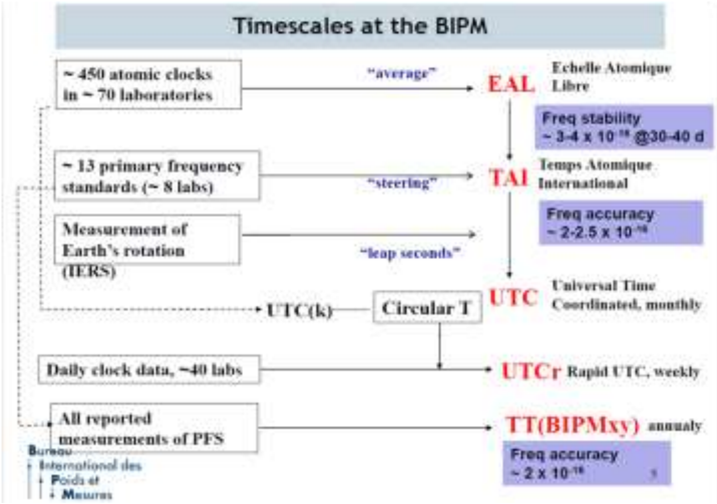


The local oscillator (RF, optical,…) is then critical to avoid to add noise and need to be narrow band enough (and low noise enough) for interrogating the physical high performance transition, and to deliver the user signal.

Clocks are more and more critical in the actual “global” world of interaction and high flow data exchange.

Time reference, UTC time constructed by BIPM from primary (optical) laboratories clocks for accuracy, and ensemble of commercial clocks for stability

The process of building up UTC “paper” scale and generating UTC(k), physical representation of the time scale UTC in every country, is depicted there:



The development of continuous improvement of primary standards and highly performing clocks, mainly driven by interrogation of trapped ion or trap atom (see EFTF , IFCS for more details) ask for long distance high performance low noise frequency comparison. Fiber link, working on distances longer # 500 km, providing an ultra low link noise are now in use.

In parallel, we can see that most of the infrastructures operating our daily lives, are driven by time scales, and are asking time dissemination or time comparison , high performance, low resolution, low jitter, down the the microsecond / nanosecond level (see pages on Telecom requirements, Renewable Energy and Smart Grid, Critical Infrastructure, IoT timing requirements… on this web site).

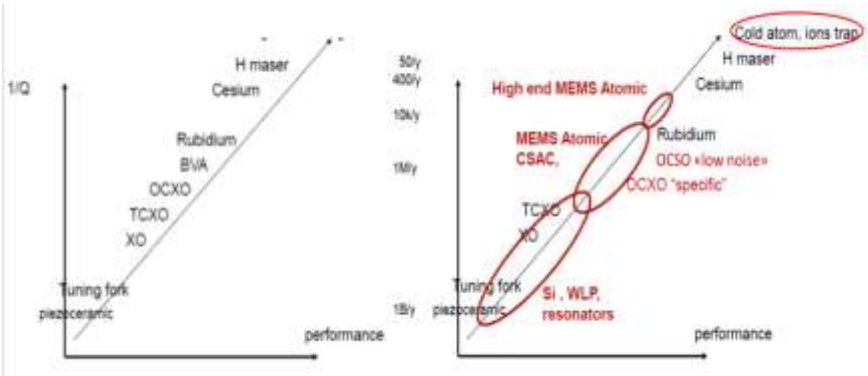
The today’s main commercially available clocks are :

- Hydrogen Maser
- Cesium clocks (optically pumped, magnetic deflexion,…)
- Rubidium (vapor lamp, laser based,…)
- DROs (Dielectric Resonator Oscillator) for frequency reference in the 1-20 GHz range
  - YIG and other high frequency specific
- Quartz (SAW devices) in frequency range 100MHz - 2000 MHz
  - MEMS reconfigurable
- Quartz thickness shear for 1-200 MHz,
- Quartz tuning fork – 32 kHz, for ultra low power (microwatt range…)

Some interesting commercial developments in terms of high performance (Cold atoms clocks or cold atoms Gravimeters) for primary clocks, Chip Scale Atomic Clock for local hold over, or optical clocks (fiber baser or WGMR based) for 30-100 GHZ sources generation, high frequency reconfigurable sources and filters, and high performance time transfer (White Rabbit PTP) are on their ways to real market.

In laboratories, there are a full range of high performance devices under development, ion trap, atom trap, optical clocks, which will probably lead to the reformulation of the legal definition of time (Actually based on electronic transition of Cesium atoms, tomorrow based on optical clocks) and fiber based frequency transfer allowing long distances comparison between these lab clocks. There are also a range of interesting developments addressing performance in miniaturized devices, one of the target being the microsecond per day in one cubic millimeter for one $, providing the required performance of local nodes in a mesh network, getting rid of base station, or allowing “all weather and condition” inertial navigation, or providing time reference for frequency agile system or Smart Grid, such as 2 D materials, atom traps, …

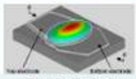
That mean that there will be a tradeoff between evolving demand of systems and infrastructure in terms of size/power/time performance and available technologies. The to day’s and to morrow clocks technologies “Trends and Evolution” might be foreseen as the following:



The recent and actual market was based quite uniquely on quartz, with a vast quantities of tuning fork low frequency (32 kHz) for watch and very low power consumption.

On clock size, one can expect a wider market coverage by Si-based or MEMS devices, at least for replacement of XOs, TCXOs and VCXOs.

On frequency reference side, such as local oscillators for metrology, for radiocommunication, DDS and PLL-LO, the “low noise” request maintain advantage towards quartz-based, while new technologies (HBAR, FBAR,..) for specific complex reconfigurable front-end devices will take the lead.

Mode of Vibration	Frequency Equation [Range]	MEMS Resonator Example
Flexural	$f_0 \propto \frac{T}{L^2}\sqrt{\frac{E}{\rho}}$ [10 KHz- 10 MHz]	 MEMS beam, Nguyen, UCB
Contour-Mode/Lamb-Wave	$f_0 \propto \frac{1}{2W}\sqrt{\frac{E}{\rho}}$ [10 MHz- 10 GHz]	 MEMS CMR, Piazza, Penn
Thickness Extensional	$f_0 \propto \frac{1}{2T}\sqrt{\frac{E}{\rho}}$ [500 MHz- 20 GHz]	 FBAR, Fujitsu
Shear Mode	$f_0 \propto \frac{1}{2T}\sqrt{\frac{G}{\rho}}$ [800 MHz -2 GHz]	 MEMS Shear Resonator, Bhave, Cornell

From G. Piaza

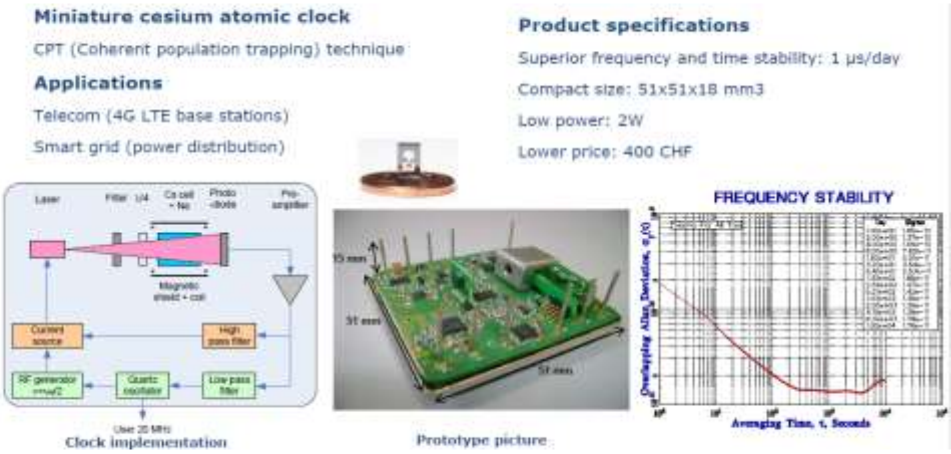
Evolution towards SDR-based (Software Defined Radio) multi-frequencies, multi-standards mobile phones drove towards frequency-agile devices, mostly filters and switched resonators, and dedicated multi purpose HW (reconfigurable front end). Filters in mobile phone are still dominated by SAW, while FBAR is commonly used for duplexers.
Si-based MEMS technology working like other acoustic devices (contour modes, flexural mode, shear-mode, thickness extension-FBAR, High overtone-HBAR…) have many technological advantages (Si-process compatible during fabrication and during board integration), further than miniaturization and integration with CMOS.

Complex system, multipurpose HW, ask for new frequency-reconfigurable plan, which, along with miniaturization, drive towards integrated MEMS technologies, such as single chip RF modules, integrated DDS, in modular front end

Other acoustic AlN (HBAR) or piezoelectric (GaPO4 for sensors, Langatate family,..)

MEMS atomic clock, mainly based on the NIST development "CSAC", under industrialization process in some countries

Based on CPT operation principle (Coherent Population Trapping), such clocks were very promising in terms of compromises between size and performances.



One can expect frequency stability in the range of some 10^{-11} frequency drift per day, or some microsecond per day. Unfortunately the glass-MEMS configuration (glass is required for transparency at Cs or Rb D1 or D2 wavelength) is still not yet stable, pollution and pressure variation inside the MEMS cell is still affecting frequency. Low power might be expected (100 mW) but high end performances are not yet “guaranteed”. Furthermore the anticipated prices are still by far too high for wide range application (see the pages “Time requirements in Telecom”, “renewable energy and time requirements in Smart Grid”, “timing in IoT world”,…).

So the “Holly Grail” of modern clocking in wide quantities of mobile devices, is still to find : 1 microsecond per day within 1 mm^3 for 1 $. There are options, but this goes out of the scope of this introduction.

Tribute of this section to JJ Gagnepain, J Vig, E Rubiola, D Howe. G Piazza, G Petit, N Dimarq, O Llopis, L Maleki, and many others